Title:
On the errors involved in the estimate of glacier ice volume from ice thickness data

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ABSTRACT
The assessment of the glacier thickness is one of the most widespread applications of radioglaciology, and is the basis for estimating the glacier volume. The accuracy of the measurement of ice thickness, the distribution of profiles over the glacier and the accuracy of the boundary delineation of the glacier are the most important factors determining the error in the evaluation of the glacier volume.

The aim of this study is to get an accurate estimate of the error incurred in the estimate of glacier volume from GPR-retrieved ice-thickness data. The errors involved can be split into *errors in boundary delineation* and *errors in computation*. The former represents the uncertainty in the definition of the glacier boundary (because of snow patches covering the terrain surrounding the glacier, debris cover, etc.), while the latter includes all the errors incurred in computing the volume once a boundary has been defined. This study focuses on the computation errors, and does not dig into the sources of the error in glacier boundary delineation.

The usual way to estimate the glacier volume is by summation of the products of cell area and average ice thickness for the cell, using a digital ice thickness model obtained by interpolation, at the grid nodes, from the ice thickness measured along the GPR profiles. Therefore, *errors in area* and *errors in thickness* both intervene in the estimate of the error in computation. For a given boundary, the inner cells have no error in area, and hence the boundary cells are the only ones contributing to the error in area, through the *pixellation error*. Since the thickness is computed, at the measurement points, as half of the product of the radio-wave velocity (RWV) and the two-way travel time, the error in the ice thickness measurements involves the errors in RWV and the errors in timing. But the ice thickness has to be interpolated, at the grid points, from the measured ice thickness (using kriging, in our case), which involves an *interpolation error*. We detail how to evaluate the latter using a function relating the error at a given grid point with the distance to the closest GPR profile. We also provide a method to evaluate the degrees of freedom in the measured data, derived from the kriging variogram, which we use for calculating the error in glacier volume by combining the errors in volume at the grid cell level.